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Technical Notes

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Laminar Film Condensation on a Thin Finite Thickness Plate

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I. Introduction

THE phenomena of laminar film condensation on vertical surfaces, taking into account nonisothermal conditions, has been analyzed in the past by numerous investigators.[1–6] In particular, Patankar and Sparrow[1] solved the problem of condensation on an extended surface by considering the heat conduction in a fin coupled with the condensation process. Subsequently, it was shown by Wilkins[2] that an explicit analytical solution is possible for the formulation of Patankar and Sparrow.[1] Sarma and Chary[3] studied the condensation process on a vertical plate fin of variable thickness. Coupling the governing equations of the vertical fin and the condensed phase, the effect of the fin geometry on the condensation heat transfer has been analyzed and the influence of this interaction is very important. Brouwers[4] performed an analysis of the condensation of a pure saturated vapor on a cooled channel plate, including the interaction between the cooling liquid, the condensate, and the vapor. He obtained in closed form the solution of the governing equations. Treviño and Méndez[5] studied the transient conjugate condensation process on one side of a vertical plate, caused by a uniform cooling rate on the other surface of the plate, including the finite thermal inertia. Their main results indicated that the condensed layer thickness evolution is almost insensitive to the longitudinal heat conduction effects through the plate for a thermally thin plate. Recently, Méndez and Treviño[6] analyzed the film condensation process of a saturated vapor in contact with one side of a vertical plate, caused by a forced flow on the other side of the plate.

In this Note, we consider the film condensation process of a saturated vapor at temperature T_s, on one lateral surface of a vertical flat plate caused by an imposed uniform temperature $T_0 < T_s$ on the other lateral surface of the plate. The effects of both longitudinal and transversal heat conduction in the plate are considered. This condensation process can be characterized mainly by two nondimensional parameters: α and ε. Parameter α represents the competition between the heat conducted longitudinally through the plate to the heat flux carried out into the plate from the condensed phase. ε is the aspect ratio of the plate (thickness-to-length ratio), assumed to be very small compared with unity. In this Note we developed an asymptotic analysis for large values of α compared with unity.

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II. Problem Formulation

A thin vertical plate of length L and thickness h is placed upright in a stagnant atmosphere filled with saturated vapor with T_s. Its upper right corner coincides with the origin of a Cartesian coordinate system whose y axis points in the normal direction to the plate, while the x axis points down in the plate's longitudinal direction, that is, in the direction of gravity g. An adiabatic wall with the same thickness h, is assumed above the leading edge and below the trailing edge. A uniform temperature $T_0 < T_s$ on the left lateral surface of the plate is imposed, generating a convective heat flux from the saturated vapor and creating a thin condensed film falling because of gravity on the right lateral surface of the plate. The condensed layer develops with increasing thickness downstream. The density of the condensed fluid ρ_c is assumed to be constant and much larger than the vapor density. The solid-condensed film governing equations have been derived elsewhere[6] and can be written as

$$\frac{\partial^2\theta_w}{\partial\chi^2} + \frac{1}{\varepsilon^2}\frac{\partial^2\theta_w}{\partial z^2} = 0, \qquad \frac{d\Delta^4}{d\chi} = \theta_w(\chi, z = 1) \tag{1}$$

with the boundary and initial conditions

$$\left.\frac{\partial\theta_w}{\partial\chi}\right|_{\chi=0,1} = 0, \qquad \theta_w(\chi, 0) = 1$$
$$\left.\frac{\partial\theta_w}{\partial z}\right|_{z=1} = -\frac{\varepsilon^2}{\alpha}\frac{\theta_w}{\Delta}, \qquad \Delta(0) = 0 \tag{2}$$

The last equation in Eq. (1) was deduced using the classical assumptions.[7–9] Here, $\chi = x/L$, $z = (y + h)/h$, $\theta_w = (T_s - T_w)/(T_s - T_0)$ (with T_w representing the temperature in the solid), and $\Delta = \delta_c\gamma^{1/4}/(LJa^{1/4})$, where δ_c represents the thickness of the condensed layer. The parameters α, ε, γ, and Ja are defined as

$$\alpha = \frac{\lambda_w}{\lambda_c}\frac{h}{L}\left(\frac{Ja}{\gamma}\right)^{1/4}, \quad \varepsilon = \frac{h}{L}, \quad \gamma = \frac{gL^3}{\nu_c^2} \quad \text{and} \quad Ja = \frac{4c_c\,\Delta T}{h_{fg}Pr_c} \tag{3}$$

Parameter α relates the competition between the heat conducted longitudinally by the plate to the heat convected to the condensed-vapor fluid. For $\alpha >> 1$, the heat conducted by the plate is very large. Thus, no temperature gradients of importance arise in the plate. In this limit the nondimensional transversal gradients of the wall temperature are very small, of order ε^2/α. This limit is called the thermally thin wall regime. On the other hand, for $\alpha \sim \varepsilon^2 << 1$ (thermally thick wall regime), the heat convected from the condensed fluid is extremely important, and the longitudinal heat conduction through the wall can be neglected. The main objective of this work is to evaluate the longitudinal heat conduction through the solid in the thermally thin wall regime ($\alpha/\varepsilon^2 >> 1$). λ_w and λ_c represent the thermal conductivity of the solid and condensed fluid, respectively; γ is a nondimensional still unnamed parameter[10]; Ja corresponds to the suitable Jakob number and represents the ratio of the sensible heat energy absorbed by the liquid to the latent heat of the liquid during condensation;

c_c is the specific heat capacity; Pr_c is the Prandtl number $Pr_c = \nu_c \rho_c c_c / \lambda_c$, of the condensed fluid; h_{fg} is the latent heat of condensation; Ku represents the Kutateladse number[11] $Ku = h_{fg}/c_c \Delta T$ with $\Delta T = T_s - T_0$; and ν_c represents the kinematic viscosity of the condensed fluid.

The overall Nusselt number is given by

$$\overline{Nu} = \int_0^1 \frac{1}{\Delta} \frac{\partial \theta_c}{\partial \eta_c}\bigg|_{\eta_c=0} \mathrm{d}\chi \equiv \frac{4}{3}\Delta^3(1) \tag{4}$$

Here, $\theta_c = (T_s - T_c)/(T_s - T_0)$, $\eta_c = y/\delta_c$, with T_c denoting the temperature in the condensed film. The most important parameter to be obtained in this work is $\Delta(1)$ or the corresponding overall Nusselt number, which is related to the total nondimensional condensing rate.

III. Asymptotic Analysis

For the thermally thin wall regime, the nondimensional temperature change in the transversal direction in the wall is very small. Thus, the longitudinal heat conduction is important, but the corresponding longitudinal temperature gradients cannot be large, because the boundary condition at the left surface of the plate is uniform. Therefore, in the asymptotic limit of $\alpha \to \infty$, the nondimensional temperature at the wall is exactly unity everywhere in the plate, and employing the second equation in Eq. (1) with $\theta_w = 1$, we obtain the classical Nusselt's solution for the nondimensional thickness of the condensed phase. In this case, the thermally thick and thin wall approximations give similar results. However, for finite values of α/ε^2, the solutions diverge slightly, as shown next.

Without inclusion of the longitudinal heat conduction through the plate, Eqs. (1) can be considerably reduced, obtaining

$$\theta_w = 1 - (1 - \theta_r)z, \qquad \theta_r = \frac{\Delta}{\varepsilon^2/\alpha + \Delta}$$
$$\Delta^4 + \frac{4}{3}\frac{\varepsilon^2}{\alpha}\Delta^3 = \chi \tag{5}$$

where θ_r represents the nondimensional temperature at the right surface of the plate. The asymptotic solution at $\chi = 1$ for large values of α/ε^2 is given by

$$\Delta(1) \sim 1 - \frac{1}{3}\frac{\varepsilon^2}{\alpha} + \mathcal{O}\left(\frac{\varepsilon^2}{\alpha}\right)^2$$
$$\overline{Nu} \sim \frac{4}{3}\left(1 - \frac{\varepsilon^2}{\alpha}\right) + \mathcal{O}\left(\frac{\varepsilon^2}{\alpha}\right)^2 \tag{6}$$

However, in the thermally thin wall regime ($\alpha/\varepsilon^2 >> 1$), the longitudinal heat conduction term must be retained in a layer of order ε in χ. The reduced inner variables in this layer can be defined by $\chi = \varepsilon\xi$, $\theta_w = 1 - (\varepsilon^{7/4}/\alpha)\varsigma$, and $\Delta = \varepsilon^{1/4}[\xi^{1/4} - (\varepsilon^{7/4}/\alpha)\Delta_1]$. The universal inner problem to be solved is then

$$\frac{\partial^2 \varsigma}{\partial \xi^2} + \frac{\partial^2 \varsigma}{\partial z^2} = 0, \qquad \frac{\mathrm{d}(\xi^{3/4}\Delta_1)}{\mathrm{d}\xi} = \frac{\varsigma}{4} \tag{7}$$

with the following boundary conditions:

$$\frac{\partial \varsigma}{\partial \xi} = 0, \quad \Delta_1 = 0 \quad \text{at} \quad \xi = 0, \quad \varsigma \to 0 \quad \text{for} \quad \xi \to \infty \tag{8}$$

$$\varsigma = 0 \quad \text{at} \quad z = 0, \qquad \frac{\partial \varsigma}{\partial z} = \frac{1}{\xi^{1/4}} \quad \text{at} \quad z = 1 \tag{9}$$

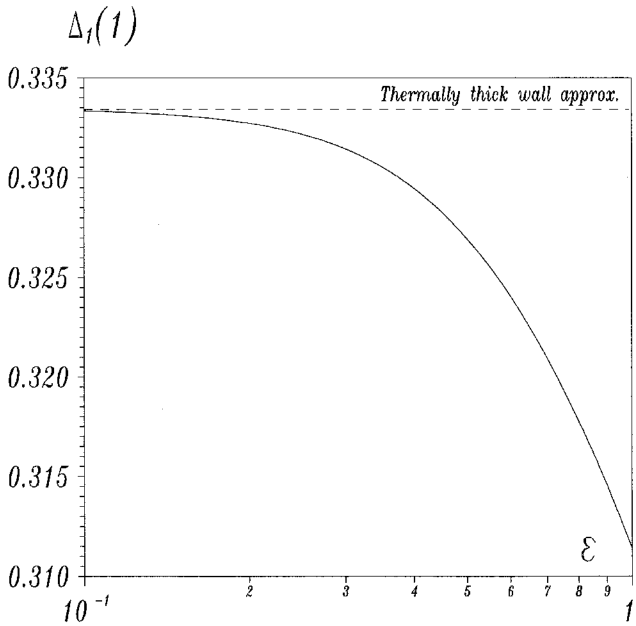

Fig. 1 Behavior of $\Delta_1(1)$ as a function of ε.

The overall reduced Nusselt in this limit takes the form

$$\overline{Nu} = \frac{4}{3}\left[1 - \frac{3}{4}\frac{\varepsilon^2}{\alpha}\Delta_1(1)\right] + \mathcal{O}\left(\frac{\varepsilon^2}{\alpha}\right)^2 \quad \text{where} \quad \Delta_1(1) = \frac{1}{4\xi^{3/4}}\int_0^{1/\varepsilon} \varsigma \, \mathrm{d}\xi \tag{10}$$

Using the Fourier cosine transform technique for Eqs. (7–9), the temperature at $z = 1$ is given by

$$\varsigma(\xi, z = 1) = \frac{2}{\pi}\Gamma\left(\frac{3}{4}\right)\cos\left(\frac{3\pi}{8}\right)\int_0^\infty \frac{\tanh(\kappa)\cos(\kappa\xi)}{\kappa^{7/4}}\mathrm{d}\kappa \tag{11}$$

where $\Gamma(m)$ represents the gamma function. For large values of ξ, the asymptotic solution is given by

$$\varsigma(\xi, z = 1) \sim (1/\xi^{1/4}) \quad \text{as} \quad \xi \to \infty \tag{12}$$

The value of $\Delta_1(\varepsilon)$ is a relatively smooth function of ε, as shown in Fig. 1. For very small values of ε, the solution tends asymptotically to that given by the thermally thick wall approximation.

IV. Conclusions

From Eqs. (6) and (10), we have almost the same dependence of ε^2/α on the value of $\Delta(1)$. However, there is a slight but finite difference for values of $\varepsilon > 0.1$. Thus, the influence of the longitudinal heat conduction, in a first approximation, is weak. The difference comes from the dependence of $\Delta_1(1)$ vs ε in the limit of the thermally thin wall regime, as seen in Fig. 1. From this figure, we conclude that for any value of ε, the Nusselt number is larger than that obtained using the thermally thick wall approximation, for fixed values of α. This means that the total amount of condensate is larger when considering the longitudinal heat conduction effects. In both cases, we recover the classical Nusselt solution of uniform temperature of the plate for $\alpha \to \infty$.

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